Q1

1

Use an appropriate method to differentiate each of the following.

(i) $\sin 2x - e^{7x}$

(ii) $x^2 \ln x$

(iii) $\dfrac{\cos 3x}{\tan 2x}$

(iv) $\ln(\tan x)$

[8]

CHAIN RULE $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$ | PRODUCT RULE $\dfrac{d(uv)}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ | QUOTIENT RULE $\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

i) $\boxed{2\cos 2x - 7e^{7x}}$ CR

ii) $u = x^2$ $\quad v = \ln x$ PR

$\dfrac{du}{dx} = 2x \quad \dfrac{dv}{dx} = \dfrac{1}{x}$

$\dfrac{dy}{dx} = \dfrac{x^2}{x} + (\ln x)2x = x + 2x\ln x = x(1+2\ln x)$

$\dfrac{dy}{dx} = \boxed{x(1+2\ln x)}$

iii) $u = \cos 3x \quad v = \tan 2x$ QR, CR

$\dfrac{du}{dx} = 3(-\sin 3x) \quad \dfrac{dv}{dx} = 2(\sec^2 2x)$

$\dfrac{dy}{dx} = \dfrac{\tan 2x(-3\sin 3x) - \cos 3x(2\sec^2 2x)}{\tan^2 2x}$

$\dfrac{dy}{dx} = \boxed{\dfrac{-3\sin 3x\tan 2x - 2\sec^2 2x\cos 3x}{\tan^2 2x}}$

iv) $\dfrac{dy}{dx} = \boxed{\dfrac{\sec^2 x}{\tan x}}$ CR

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Q2

2

A curve has the equation $y = e^{-3x} + \ln x,\ x > 0$.

Show that the equation of the tangent to the curve at the point with x-coordinate 1 is

$$y = \left(\frac{e^3 - 3}{e^3}\right)x + \frac{4 - e^3}{e^3}$$

[6]

$y - y_1 = m(x - x_1)$

When $x = 1$

$y = e^{-3(1)} + \ln 1 = e^{-3}$

$m = \dfrac{dy}{dx} = -3e^{-3x} + \dfrac{1}{x}$

When $x = 1$

$m = -3e^{-3(1)} + \dfrac{1}{(1)} = -3e^{-3} + 1$

$y - e^{-3} = (-3e^{-3} + 1)(x - 1)$

$y = -3e^{-3}x + 3e^{-3} + x - 1 + e^{-3}$

$= (-3e^{-3} + 1)x + (4e^{-3} - 1)$

$= \left(\dfrac{-3}{e^3} + \dfrac{e^3}{e^3}\right)x + \left(\dfrac{4}{e^3} - \dfrac{e^3}{e^3}\right)$

$\boxed{y = \left(\dfrac{-3 + e^3}{e^3}\right)x + \left(\dfrac{4 - e^3}{e^3}\right)}$

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Q3

3

For $y = \ln(ax^n)$, where $a > 0$ is a real number and $n \geq 1$ is an integer, show that

$$\frac{dy}{dx} = \frac{n}{x}$$

[3]

Method 1: Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Let $y = \ln u$ $\quad u = ax^n$

$\frac{dy}{du} = \frac{1}{u}$ $\quad \frac{du}{dx} = anx^{n-1}$

$$\frac{dy}{dx} = \frac{1}{u}\, anx^{n-1}$$

$$= \frac{\cancel{a}nx^{n-1}}{\cancel{a}x^n} = nx^{-1} = \frac{n}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{n}{x}}$$

OR

Method 2: Laws of Logs

$$y = \ln(ax^n) = \ln a + \ln x^n = \ln a + n\ln x$$

$$\boxed{\frac{dy}{dx} = \frac{n}{x}}$$

Q4

4

Find the gradient of the normal to the curve $y = 5\cos(e^x - \frac{\pi}{2})$ at the point with x-coordinate 0. Give your answer correct to 3 decimal places.

[4]

$$m_{normal} = \frac{-1}{m_{tangent}} = \frac{-1}{\frac{dy}{dx}}$$

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Let $y = 5\cos u$ $\quad u = e^x - \frac{\pi}{2}$

$\frac{dy}{du} = -5\sin u$ $\quad \frac{du}{dx} = e^x$

$$\frac{dy}{dx} = -5(\sin u)e^x$$

$$= -5\left(\sin\left(e^x - \frac{\pi}{2}\right)\right)e^x$$

Sub in $x = 0$

$$\frac{dy}{dx} = -5\left(\sin\left(e^0 - \frac{\pi}{2}\right)\right)e^0$$

$$= -5\sin\left(1 - \frac{\pi}{2}\right)$$

$$= 2.70...$$

$$m_{normal} = \frac{-1}{2.70...} = \boxed{-0.370} \quad (3\text{ dp})$$

Q5a

5a

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(2\sin 3x - \cos 3x)e^{6-x}$ [3]

(b) $(x^2 - x)^2 \ln 5x$ [3]

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a)

Product rule $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Let $u = 2\sin 3x - \cos 3x$ $\quad v = e^{6-x}$

$\frac{du}{dx} = 6\cos 3x + 3\sin 3x$ $\quad \frac{dv}{dx} = -e^{6-x}$

$$\frac{d(uv)}{dx} = (2\sin 3x - \cos 3x)(-e^{6-x}) + (e^{6-x})(6\cos 3x + 3\sin 3x)$$

$$= e^{6-x}(-2\sin 3x + \cos 3x + 6\cos 3x + 3\sin 3x)$$

$$= \boxed{e^{6-x}(\sin 3x + 7\cos 3x)}$$

Q5b

5b

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(2\sin 3x - \cos 3x)e^{6-x}$ [3]

(b) $(x^2 - x)^2 \ln 5x$ [3]

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b)

Product rule $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Let $u = (x^2 - x)^2$ $\quad v = \ln 5x$

$= x^4 - 2x^3 + x^2$ $\quad = \ln 5 + \ln x$

$\frac{du}{dx} = 4x^3 - 6x^2 + 2x$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$$\frac{d(uv)}{dx} = \frac{(x^2 - x)^2}{x} + (\ln 5x)(4x^3 - 6x^2 + 2x)$$

$$= \frac{x^4 - 2x^3 + x^2}{x} + (\ln 5x)(4x^3 - 6x^2 + 2x)$$

$$= \boxed{x^3 - 2x^2 + x + (\ln 5x)(4x^3 - 6x^2 + 2x)}$$

Q6

6

By writing $y = \frac{f(x)}{g(x)}$ as $y = f(x)[g(x)]^{-1}$ and then using the product and chain rules, show that

$$\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

[3]

Product rule $\qquad \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$y = uv$

Let $u = f(x) \qquad v = (g(x))^{-1}$

chain rule $(u = g(x))$ $\qquad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{du}{dx} = f'(x) \qquad \frac{dv}{dx} = (g'(x))(-g(x)^{-2})$

$\frac{dy}{dx} = \frac{d(uv)}{dx} = f(x)g'(x)(-(g(x))^{-2}) + (g(x))^{-1}f'(x)$ $\quad \times \frac{g(x)}{g(x)}$

$= \frac{-f(x)g'(x)}{(g(x))^2} + \frac{f'(x)g(x)}{(g(x))^2}$

$$\boxed{\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}}$$

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Q7a

7a

Given that $x = \sec 7y$,

(a) Find $\frac{dy}{dx}$ in terms of y [2]

$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

(b) Hence find $\frac{dy}{dx}$ in terms of x. [4]

a) $x = \frac{1}{\cos 7y} = (\cos 7y)^{-1}$ $\qquad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Chain rule

flip around the fractions OR exchange the xs and ys

$\frac{dx}{dy} = \frac{dx}{du} \times \frac{du}{dy}$

Let $x = u^{-1} \qquad u = \cos 7y$

$\frac{dx}{du} = -u^{-2} \qquad \frac{du}{dy} = -7\sin 7y$

$\frac{dx}{dy} = -u^{-2}(-7\sin 7y) = \frac{7\sin 7y}{u^2}$

$= \frac{7\sin 7y}{(\cos 7y)^2} = 7\tan 7y \sec 7y$

$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{7\tan 7y \sec 7y}$

$$\boxed{\frac{dy}{dx} = \frac{1}{7\tan 7y \sec 7y}}$$

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Q7b

7b

Given that $x = \sec 7y$,

(a) Find $\frac{dy}{dx}$ in terms of y **[2]**

$$\frac{dy}{dx} = \frac{1}{7\tan 7y \sec 7y}$$

$\tan 7y \rightarrow \sqrt{x^2+1}$, $\sec 7y \rightarrow x$

(b) Hence find $\frac{dy}{dx}$ in terms of x. **[4]**

b) $x = \sec 7y$

Find $\tan 7y$ in terms of x

$\sec^2 7y = \tan^2 7y - 1$ trig identity!

$x^2 = \tan^2 7y - 1$

$x^2 + 1 = \tan^2 7y$

$\sqrt{x^2+1} = \tan 7y$

$$\boxed{\frac{dy}{dx} = \frac{1}{7x\sqrt{x^2+1}}}$$

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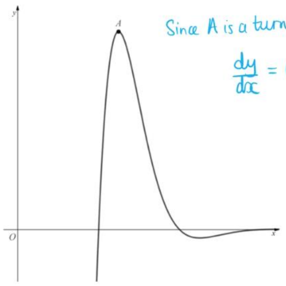
Q8

8

The diagram below shows part of the graph of $y = f(x)$, where $f(x)$ is the function defined by

$$f(x) = \frac{\sin x}{1 - e^x}, \quad x > 0$$



Since A is a turning point:

$\frac{dy}{dx} = 0$

Point A is a maximum point on the graph.

Show that the x-coordinate of A is a solution to the equation

$$\frac{\cos x + e^x(\sin x - \cos x)}{e^{2x} - 2e^x + 1} = 0$$

[5]

Quotient Rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Let $y = \frac{u}{v}$

$u = \sin x \qquad v = 1 - e^x$

$\frac{du}{dx} = \cos x \qquad \frac{dv}{dx} = -e^x$

$$\frac{dy}{dx} = \frac{(1-e^x)\cos x - (\sin x)(-e^x)}{(1-e^x)^2} = 0$$

$$\frac{\cos x - e^x\cos x + e^x \sin x}{1 + e^{2x} - 2e^x} = 0$$

$$\boxed{\frac{\cos x + e^x(\sin x - \cos x)}{e^{2x} - 2e^x + 1} = 0}$$

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